Newchurch Community Primary School

Mathematics Policy and Building to written methods

Linked Policy Documents:

- Visual Calculation Policy
- Visual Fractions Policy
- Marking and Feedback Policy

Contents:

Introduction

The Mathematics framework provides a structured and systematic approach to teaching number. There is a considerable emphasis on teaching mental calculation strategies and speaking and listening activities. Up to the age of 9 (Year 4) informal written recording should take place regularly and is an important part of learning and understanding. **More formal written methods should follow only when the child is able to use a wide range of mental calculation strategies.** This will help communicate methods and solutions.

Why do we need this policy?

- Consistency in methods taught throughout the school.
- Progression from informal / practical methods of recording to written methods for each of the four operations.
- An aid to parent's understanding in their child's stages of learning.

Reasons for using written methods

- To aid mental calculation by writing down some of the numbers and answers involved
- To make clear a mental procedure for the pupil
- To help communicate methods and solutions
- To provide a record of work to be done
- To aid calculation when the problem is too difficult to be done mentally
- To develop and refine a set of rules for calculation

How mathematics is taught at Newchurch:

The aim of the mathematics approach is to develop the children's mental calculation confidence before moving onto the written methods of formal mathematics. The lessons will be differentiated to meet the needs of the children, however they will work within the expectations of the National Curriculum.

The children will meet mathematics in three main formats:

1. Fluency – This is be the children's ability to perform the base standard of the target e.g. perform a written calculation method.
2. Reasoning - The children will apply their knowledge of number and methods to more contextual problems including word problems.
3. Problem solving - The children will investigate more expansive challenges which employ their mathematics knowledge. This can include open-ended tasks and those linked to other areas of the curriculum e.g. mathematics within science.

Marking and Feedback will support the children in progressing between these three stages. They will be supported in their learning through the use of concrete manipulatives (objects), visual support (images) and finally abstract methodology.

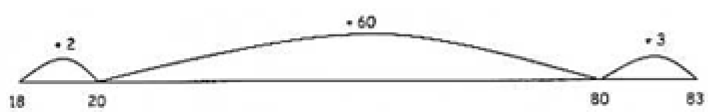
Whole school approach

We have developed a consistent approach to the teaching of written calculation methods. This will establish continuity and progression throughout the school.

Different mental methods will be established in Key Stage 1 and built on as the children progress into Key Stage 2. These are shown below and will be based on a solid understanding of place value in number.

Things to remember for Key Stage One

i. Remembering number facts and recalling them without hesitation e.g. pairs of numbers that make 10
ii. Doubles and halves to 20
iii. Using known facts to calculate unknown facts
e.g. 6 + 6 = 12 therefore 6 + 7 = 13
24 + 10 = 34 therefore 24 + 9 = 33
iv. Understanding and using relationships between addition and subtraction to find answers and check results
e.g. 14 + 6 = 20 therefore 20 – 6 = 14
v. Having a repertoire of mental strategies to solve calculations
e.g. 14 + 6 =20 therefore 20 – 6 = 14
bridging 10 / bridging 20
adding 9 by +10 & -1
vi. Making use of informal jottings such as blank number lines to assist in calculations with larger numbers e.g. 83 – 18 = 65



vii. Solving one-step word problems (either mentally of with jottings) by identifying which operation to use, drawing upon knowledge of number bonds and explaining their reasoning
viii. Beginning to present calculations in a horizontal format and explain mental steps using numbers, symbols or words
ix. Learn to estimate/approximate first e.g. 29 + 30 (round to the nearest 10, the answer will be near 60)

Place value will be taught by counting on and counting back depending on the numbers.

Numbers such as 10, 100 and 1000 will be called Landmark Numbers.

When are children ready for written calculation?

Addition and Subtraction

- Do they know addition and subtraction facts to 20?
- Do they know place value and can they partition numbers in a variety of ways? E.g. 12 = 10 + 2, 12 = 9 + 3 12 = 8 + 4
- Can they add three single digit numbers mentally?
- Can they add and subtract any pair of two digit numbers mentally?
- Can they explain their mental strategies orally and record them using informal jottings?

Multiplication and Division

- Do they know their 2,3,4,5 and 10 time tables?
- Do they know the result of multiplying by 0 and 1?
- Do they understand 0 as a placeholder?
- Can they multiply two and three digit numbers 10 and 100?
- Can they double and halve two digit numbers mentally?
- Can they use multiplication facts they know to derive mentally other multiplication facts that they do not now
- Can they explain their mental strategies orally and record them using informal jottings?

The above lists are not exhaustive but are a guide for the teacher to judge when a child is ready to move from informal to formal methods of calculation.

Stages in Addition – (Please refer to the Visual Calculation for a more detailed breakdown)

1. Mental method, using partitioning:

   47 +76 = (40 + 70) + (7 + 6)

   Or

   47 + 76 = (47 + 70) + 6

2. Introduction to vertical layout, using partitioning:

$$\begin{array}{rcrcr} 300 & + & 70 & + & 8 \\ \underline{400} & \underline{+} & \underline{80} & \underline{+} & \underline{7} \\ \underline{700} & \underline{+} & \underline{150} & \underline{+} & \underline{15} \\ & = & 865 & & \end{array}$$

3. Vertical layout, expanded working, adding the least significant digit first:

$$\begin{array}{lr} & 47 \\ + & \underline{76} \\ & 13 \\ & \underline{110} \\ & 123 \end{array} \qquad \begin{array}{lr} & 368 \\ + & \underline{493} \\ & 11 \\ & 150 \\ & \underline{700} \\ & \underline{861} \end{array}$$

4. Vertical layout, contracting the working to compact efficient form:

$$\begin{array}{lr} & 47 \\ + & \underline{76} \\ & 13 \\ & \underline{110} \\ & 123 \end{array} \qquad \begin{array}{lr} & 47 \\ + & 76 \\ & \underline{{}^{1\,1}} \\ & \underline{123} \end{array} \qquad \begin{array}{lr} & 368 \\ + & \underline{493} \\ & 11 \\ & 150 \\ & \underline{700} \\ & \underline{861} \end{array} \qquad \begin{array}{lr} & 368 \\ + & 493 \\ & \underline{{}^{1\,1}} \\ & \underline{861} \end{array}$$

5. Moving on to larger numbers and decimals, before moving onto more abstract forms such as algebra and fractions.

Stages in Subtraction – (Please refer to the Visual Calculation for a more detailed breakdown)

1. Methods using decomposition

89 – 65

$$\begin{array}{lrr} & 80 & 9 \\ - & \underline{60} & \underline{5} \\ & \underline{20} & \underline{4} = \quad 24 \end{array}$$

563 -241

$$\begin{array}{lrrr} & 500 & 60 & 3 \\ - & \underline{200} & \underline{40} & \underline{1} \\ & \underline{300} & \underline{20} & \underline{2} = \quad 322 \end{array}$$

Leading to:

$$\begin{array}{lr} & 89 \\ - & \underline{65} \\ & \underline{24} \end{array} \qquad \begin{array}{lr} & 563 \\ - & \underline{241} \\ & \underline{322} \end{array}$$

2. Vertical layout using expanded partitioning:

85 – 69

$$\begin{array}{lrr} & 70 & 15 \\ - & \underline{60} & \underline{9} \\ & \underline{10} & \underline{6} = \quad 16 \end{array}$$

523 - 244

$$\begin{array}{lrrr} & 400 & 110 & 13 \\ - & \underline{200} & \underline{40} & \underline{4} \\ & \underline{200} & \underline{70} & \underline{9} = \quad 279 \end{array}$$

3. Using vertical layout, contracting the working moving to a compact efficient form:

85 - 69

$$\begin{array}{lr} & {}^{7}\cancel{8}{}^{1}5 \\ - & \underline{69} \\ & \underline{16} \end{array}$$

563 -278

$$\begin{array}{lr} & {}^{4}\cancel{5}{}^{15}\cancel{6}{}^{1}3 \\ - & \underline{2\ 7\ 8} \\ & \underline{2\ 8\ 5} \end{array}$$

Stages in Multiplication

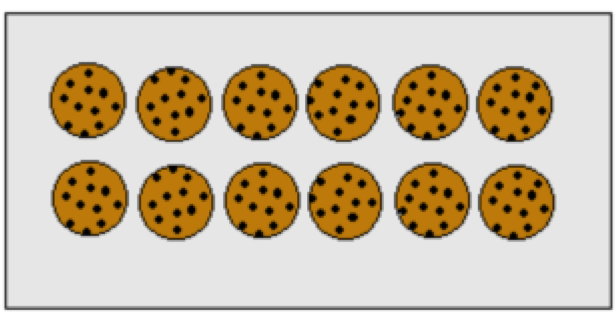
To multiply successfully, children need to be able to:

- recall all multiplication facts to 10 × 10
- partition number into multiples of one hundred, ten and one
- work out products such as 70 × 5, 70 × 50, 700 × 5 or700 × 50 using the related fact 7 × 5 and their knowledge of place value
- add two or more single-digit numbers mentally
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value
- add combinations of whole numbers using the column method (see above).

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication.

1. Arrays. Children can start in Key Stage 1 to understand the concept of multiplications by using arrays. Arrays can help your children develop concepts of multiplication and division.

The teacher will say, "An array shows objects in rows and columns. The teacher will show an example of a row and column using an array illustration in this case cookies on a cookie sheet. (2 x 6 = 12)



2. Repeated Addition:

You know, a cat has four legs. Suppose there are 3 cats. How many legs are there altogether?

The answer is 4 + 4 + 4. Using our knowledge of addition, we can find this repeated addition as 4 + 4 + 4 = 12
or 3 times 4 is 12 or 3 x 4 = 12.

3. Mental Method using partitioning, multiplying tens first:

   38 x 7 = (30 x 7) + (8 x 7) = 210 + 56 = 266

4. Grid Layout Method (2 digit by 1 digit):

X	30	8
7	210	56

$$\begin{array}{r} 210 \\ +\ 55 \\ \hline 266 \end{array}$$

5. Grid Layout Method (3 digit by 1 digit) i.e. 238 x 7:

X	200	30	8
7	1400	210	56

$$\begin{array}{r} 1400 \\ 210 \\ +\ 56 \\ \hline 1666 \end{array}$$

6. Grid Layout extended to bigger numbers (ThHTU)

i.e. 56 x 27 = (50 + 6) x (20 + 7)

X	50	6
20	1000	120
7	350	42

$$\begin{array}{r} 1120 \\ 392 \\ \hline 1512 \end{array}$$

7. Vertical Format, expanded working:

$$\begin{array}{rr} & 38 \\ \times & 7 \\ \hline & 210 \\ & 56 \\ \hline & 266 \end{array}$$

8. Extended to HTU x U

Long Multiplication

$$\begin{array}{rrl} & 56 & \\ \times & 27 & \\ \hline & 1000 & (50 \times 20) \\ & 120 & (6 \times 20) \\ & 350 & (50 \times 7) \\ & 42 & (6 \times 7) \\ \hline & 1512 & \end{array}$$

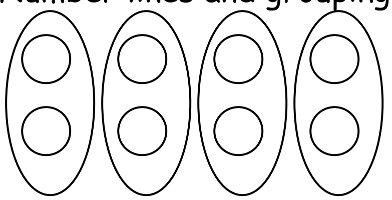
9. Vertical Format, compact working:

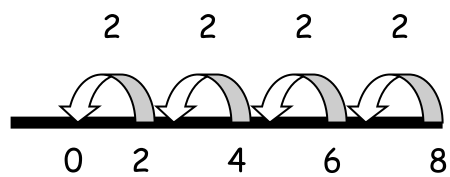
$$\begin{array}{rl} 56 & \\ 27 & \\ \hline 1120 & (56 \times 20) \\ 392 & (56 \times 7) \\ \hline 1512 & \\ \hline {}_{1} & \end{array}$$

Stages in Division

To introduce division it should be practical, using equipment to demonstrate. Children in Key Stage 1 are to understand division as grouping and sharing which is repeated subtraction

1. Number lines and grouping:





2. Informal methods using multiples of the divisor or 'chunking' TU ÷ U:

72 ÷ 5	72			
	- 50	50 =	10	x 5
	22			
	- 20	20 =	4	x 5
	2			
			14	

Answer: 14 r 2

3. 'Chunking' HTU ÷ U

256 ÷ 7	256			
	- 70	70 =	10	x 7
	186			
	- 140	140 =	20	x 7
	46			
	- 42	42 =	6	x 7
	4		**36**	

Answer: 36 r 4

4. Efficient 'Chunking' HTU ÷ U

196 ÷ 6	196			
	- 180	180 =	30	x 6
	16			
	- 12	12 =	2	x 6
	4		**32**	

<u>Answer: 32 r 4</u>

5. Extending to decimals with up to 1 place

87.5 ÷ 7	87.5			
	- 70.0	70 =	10	x 7
	17.5			
	- 14.0	14 =	2	x 7
	3.5			
	- 3.5	3.5 =	0.5	x 7
	0		**12.5**	

<u>Answer: 12.5</u>

6. Chunking Extended HTU ÷ TU (Efficiently developed):

560 ÷ 24	560			
	- 480	480 =	20	x 24
	80			
	- 72	72 =	3	x 24
	8		**23**	

<u>Answer: 23 r 8</u>

7. Extending to an efficient standard method:

560 ÷ 24

```
            ___
24         )560
            ___
          - 480      |  20  |
             80      |      |
                     |      |
            -72      |   3  |
            ___
              8         23
```

<u>Answer</u>: 23 r 8

8. Partitioning standard method:

560 ÷ 24

```
       ___________
24    )560

        10 +   10 +   3 r 8
24    _______________________
       240 + 240 + 80
```

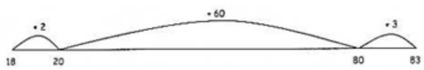
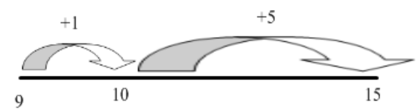
<u>Answer</u>: 23 r 8

9. Extend to Compact Method

560 ÷ 24

```
          2 3 r 8
        _________
24     )5̶ 5 6̶ 86 0
```

<u>Answer</u>: 23 r 8

	Addition	Subtraction
Y3	TU +TU developing to HTU +TU or HTU + HTU 1. Use number lines to count on.  2. Horizontal expanded method, using partitioning. 47 +76 = (40 + 70) + (7 + 6) *Or* 47 + 76 = (47 + 70) + 6 3. Vertical expanded method adding least significant digit first. 47 + 76 13 110 123	TU – TU, developing to HTU –TU or HTU – HTU. 1. Use a number line to count up. 15 - 9  2. Decomposition using expanded form 89 – 65 80 9 - 60 5 20 4 = 24 3. Use vertical form (expanded partitioning) 85 – 69 70 15 - 60 9 10 6 = 16
Y4	HTU + TU then HTU + HTU 1. Vertical expanded method adding least significant digit first. 47 + 76 13 110 123	HTU – TU then HTU – HTU 1. Decomposition using expanded form. 89 – 65 80 9 - 60 5 20 4 = 24 2. Decomposition using compact form.

	Addition	Subtraction
	2. Leading to 'carrying' above the line. 368 + 93 11 461 3. Calculations extending to include addition of two or more 3-digit sums of money. £3.68 + 93 11 £4.61	85 – 69 7~~8~~15 - 69 16 3. Calculations extending to include addition of two or more 3-digit sums of money. £2~~3~~.168 - 1. 93 £ 1.75
Y5	HTU + HTU then ThHTU + ThHTU 1. Vertical expanded method adding least significant digit first. 1,356 + 2,487 13 130 700 3000 3843 2. Leading to compact written method 'carrying' above the line. 1,356 2,487 11 3843 3. Calculations extended to include addition of two of more decimal fractions, with up to 3 digits and same number of decimal places, in vertical format	HTU – HTU the ThHTU – ThHTU . Decomposition using expanded form. 189 – 165 100 80 9 - 100 60 5 0 60 4 = 24 2. Decomposition using compact form. 185 – 169 1 7~~8~~15 - 169 16 3. Calculations extending to include subtraction of decimals, with up to 3 digits & and the same number of decimal places, in expanded format leading to vertical format.
Y6	Th HTU + ThHTU & then any number of digits. 1. Compact written method 'carrying' above the line. 1,356 2,487 1 3843	ThHTU – THHTU & then any number of digits 1. Decomposition using compact form. 1~~2~~,12~~3~~145 1 ,7 65 5 80

<table>
<tr><th></th><th>Addition</th><th>Subtraction</th></tr>
<tr><td></td><td>2. Calculations extended to include addition of two or more decimal fractions with up to for digits & either one or two decimal places.</td><td>2. Calculations extended to include subtraction of two or more decimal fractions with up to 3 digits & either one or two decimal places in vertical format.</td></tr>
</table>

<table>
<tr><th></th><th>Multiplication</th><th>Division</th></tr>
<tr><td>KS1</td><td>Arrays and Repeated Addition</td><td>Arrays, Number Lines Grouping and Sharing</td></tr>
<tr><td>Y3</td><td>Mental Method using partitioning, multiplying tens first:

38 x 7 = (30 x 7) + (8 x 7)
= 210 + 56 = 266

Grid Layout Method (2 digit by 1 digit):

<table>
<tr><td>X</td><td>30</td><td>8</td><td></td><td>210</td></tr>
<tr><td>7</td><td>210</td><td>56</td><td></td><td>+ 56</td></tr>
<tr><td></td><td></td><td></td><td></td><td>266</td></tr>
</table></td><td>Informal methods using multiples of the divisor or ‘chunking’ TU ÷ U:

<table>
<tr><td>72 ÷ 5</td><td>72</td><td></td><td></td><td></td></tr>
<tr><td></td><td>- 50</td><td>50 =</td><td>10</td><td>x 5</td></tr>
<tr><td></td><td>22</td><td></td><td></td><td></td></tr>
<tr><td></td><td>- 20</td><td>20 =</td><td>4</td><td>x 5</td></tr>
<tr><td></td><td>2</td><td></td><td>14</td><td></td></tr>
</table></td></tr>
<tr><td>Y4</td><td>Grid Layout Method (3 digit by 1 digit)
i.e. 238 x 7:

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<tr><td>7</td><td>1400</td><td>210</td><td>56</td><td>1400</td></tr>
<tr><td></td><td></td><td></td><td></td><td>210</td></tr>
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<tr><td></td><td></td><td></td><td></td><td>1666</td></tr>
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i.e 256 ÷ 7

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<tr><td>256 ÷ 7</td><td>256</td><td></td><td></td><td></td></tr>
<tr><td></td><td>- 70</td><td>70 =</td><td>10</td><td>x 7</td></tr>
<tr><td></td><td>186</td><td></td><td></td><td></td></tr>
<tr><td></td><td>- 140</td><td>140 =</td><td>20</td><td>x 7</td></tr>
<tr><td></td><td>46</td><td></td><td></td><td></td></tr>
<tr><td></td><td>- 42</td><td>42 =</td><td>6</td><td>x 7</td></tr>
<tr><td></td><td>4</td><td></td><td>36</td><td></td></tr>
</table></td></tr>
</table>

<table>
<tr><td>Y5</td><td>Grid Layout extended to bigger numbers (ThHTU)

i.e. 56 x 27 = (50 + 6) x (20 + 7)

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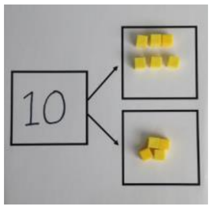
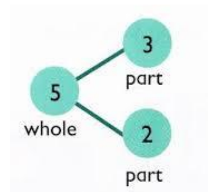
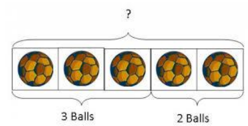
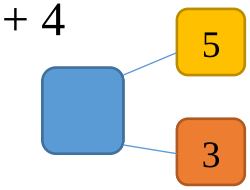
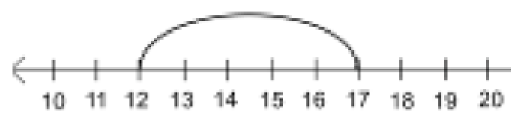
<tr><td>X</td><td>50</td><td>6</td></tr>
<tr><td></td><td></td><td></td></tr>
</table></td><td>Extending to decimals with up to 1 place

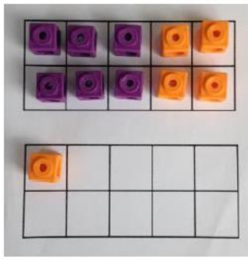
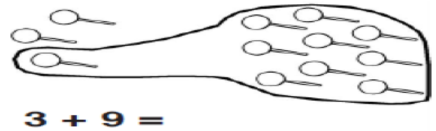
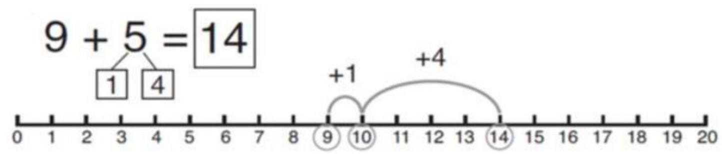
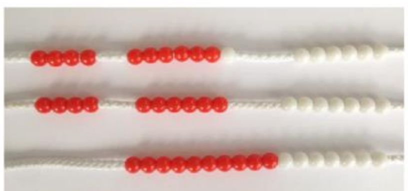
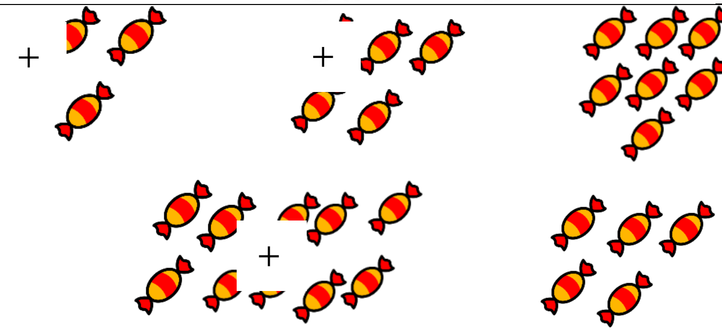
<table>
<tr><td>87.5 ÷ 7</td><td>87.5</td><td></td><td></td><td></td></tr>
<tr><td></td><td>- 70.0</td><td>70 =</td><td>10</td><td>x 7</td></tr>
<tr><td></td><td>17.5</td><td></td><td></td><td></td></tr>
<tr><td></td><td>- 14.0</td><td>14 =</td><td>2</td><td>x 7</td></tr>
<tr><td></td><td>3.5</td><td></td><td></td><td></td></tr>
<tr><td></td><td>- 3.5</td><td>3.5 =</td><td>0.5</td><td>x 7</td></tr>
</table></td></tr>
</table>

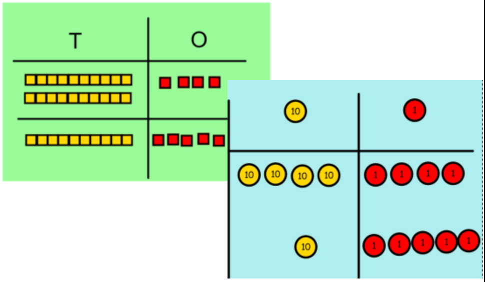
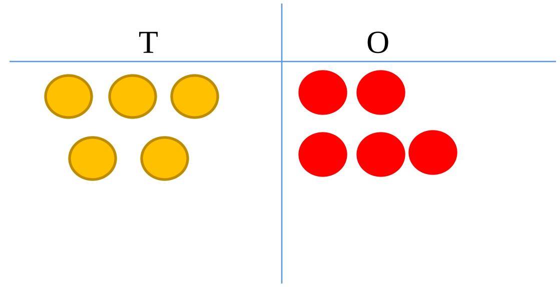
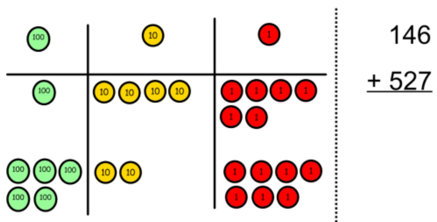
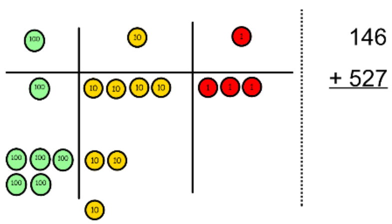
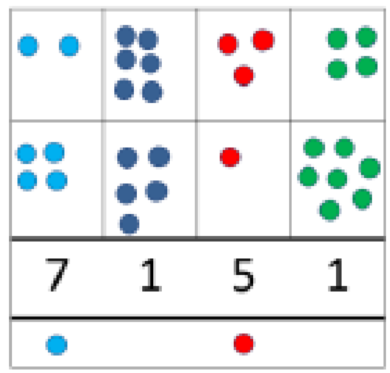
	20 1000 120 7 350 42	**0** **12.5** Chunking Extended HTU ÷ TU (Efficiently developed): 560 ÷ 24 560 - 480 480 = 20 x 24 80 - 72 72 = 3 x 24 **8** **23**
Y6	Vertical Format, expanded working: 38 X 7 210 56 266 Extended to HTU x U Long Multiplication 56 X 27 1000 (50 x 20) 120 (6 x 20) 350 (50 x 7) 42 (6 x 7) 1512 Vertical Format, compact working: 56 X 27 1120 (56 x 20) 392 (56 x 7) 1512 1	Extending to an efficient standard method: 560 ÷ 24 24)560 - 480 20 80 -72 3 **8** **23** Partitioning standard method: 560 ÷ 24 24)560 ~~10 + 10 + 3 r 8~~ 24 240 + 240 + 80 Extend to Compact Method 2 3 r 8 560 ÷ 24 24)~~5~~5~~6~~860

Progression in Calculations

Addition

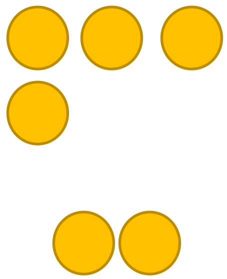
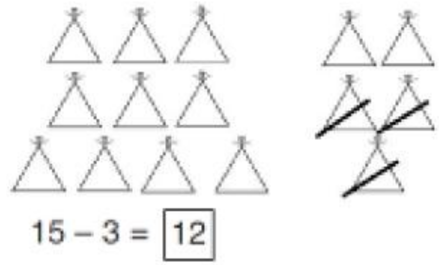
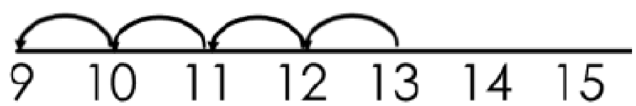
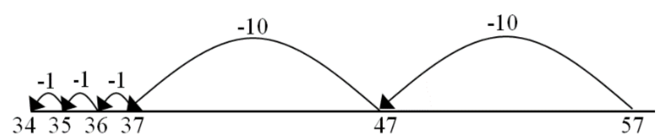
Objective and Strategies	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model	  Use cubes to add two numbers together as a group or in a bar. 	   Use pictures to add two numbers together as a group or in a bar.  	4 + 3 = 7 10= 6 + 4  Use the part-part whole diagram as shown above to move into the abstract.
Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.

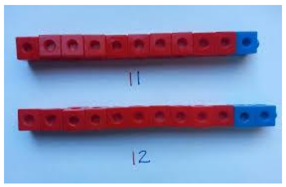
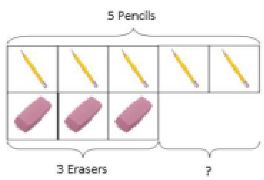
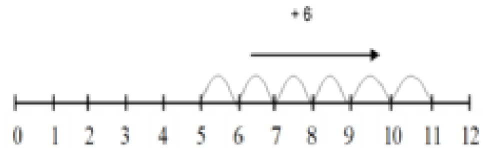
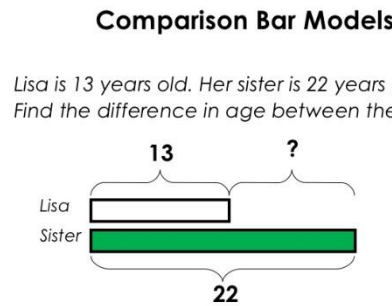
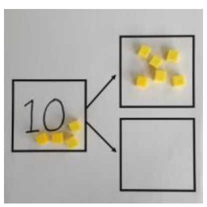
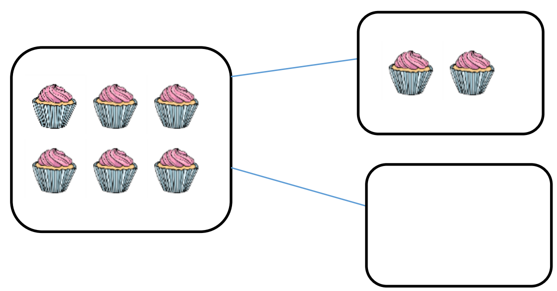
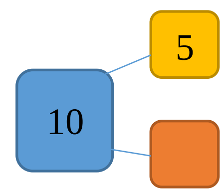
Regrouping to make 10.	 6 + 5 = 11  Start with the bigger number and use the smaller number to make 10.	 Use pictures or a number line. Regroup or partition the smaller number to make 10.  	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now?
Adding three single digits	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	$4 + 7 + 6 = \boxed{10} + \boxed{7}$ 10 $= \boxed{17}$ Combine the two numbers that make 10 and then add on the remainder.

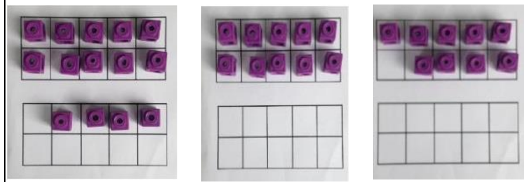
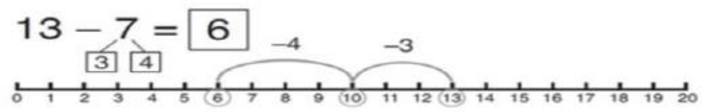
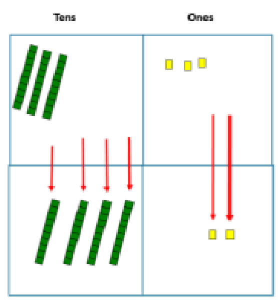
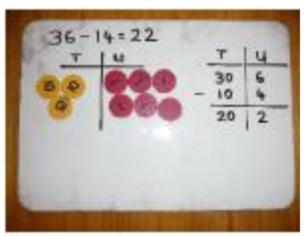
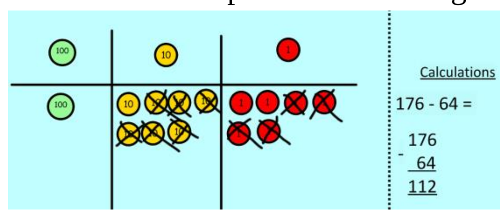
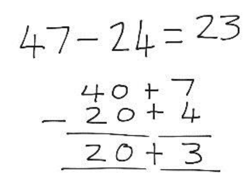
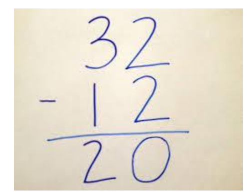
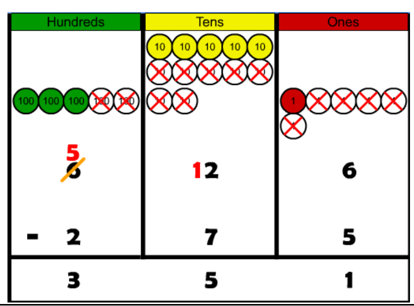
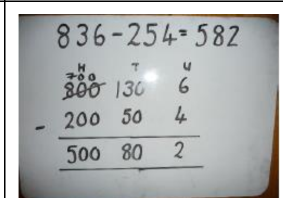
Column method- no regrouping	24 + 15= Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.  	After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.  	Calculations 21 + 42 = 21 + 42
Column method- regrouping	Make both numbers on a place value grid.  Add up the units and exchange 10 ones for one 10.  Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.	Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding.  	Start by partitioning the numbers before moving on to clearly show the exchange below the addition. 20 + 5 40 + 8 60 + 13 = 73 536 + 85 621 1 1

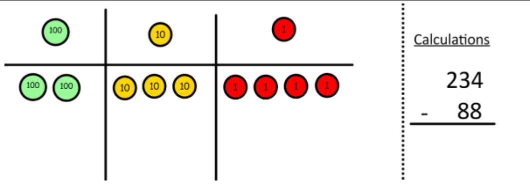
	This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100. As children move on to decimals, money and decimal place value counters can be used to support learning.		As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here. 72.8 + 54.6 127.4 1 1 £ 2 3 . 5 9 + £ 7 . 5 5 £ 3 1 . 1 4 1 1 1 2 3 . 3 6 1 9 . 0 8 0 5 9 . 7 7 0 + 1 . 3 0 0 9 3 . 5 1 1 2 1 2

Subtraction

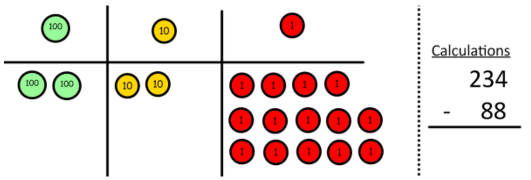
Objective and Strategies	Concrete	Pictorial	Abstract
Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away.  6 – 2 = 4 	Cross out drawn objects to show what has been taken away. 	18 -3= 15 8 – 2 = 6
Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.  13 – 4 Use counters and move them away from the group as you take them away counting backwards as you go. 	Count back on a number line or number track  Start at the bigger number and count back the smaller number showing the jumps on the number line.  This can progress all the way to counting back using two 2 digit numbers.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.

Find the difference	Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference  Use basic bar models with items to find the difference	 Count on to find the difference. Draw bars to find the difference between 2 numbers. 	Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.
Part Part Whole Model	 Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the part part whole model. 	 Move to using numbers within the part whole model.

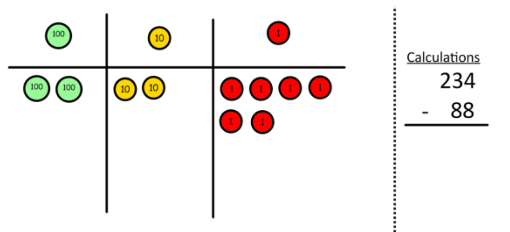
Make 10	14 – 9 =  Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.	16 – 8= How many do we take off to reach the next 10? How many do we have left to take off?
Column method without regrouping	 Use Base 10 to make the bigger number then take the smaller number away. Show how you partition numbers to subtract. Again make the larger number first.  	 Draw the Base 10 or place value counters alongside the written calculation to help to show working.  	 This will lead to a clear  written column subtraction.
Column method with regrouping	Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges. Make the larger number with the place value counters	 	



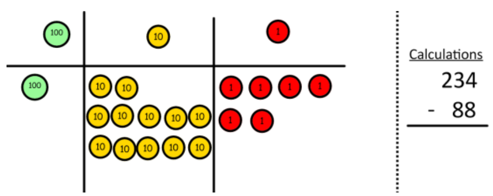
Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.



Now I can subtract my ones.



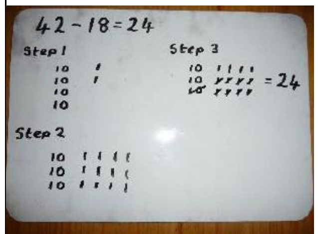
Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.



Now I can take away eight tens and complete my subtraction

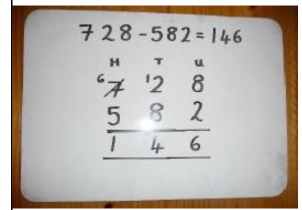
Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.

When confident, children can find their own way to record the exchange/regrouping.



Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.

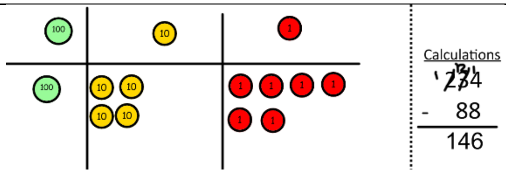
Children can start their formal written method by partitioning the number into clear place value columns.



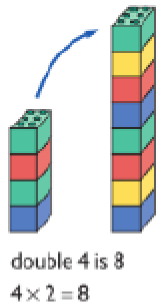
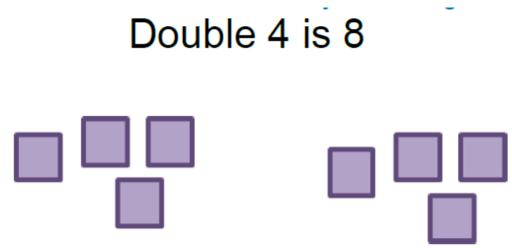
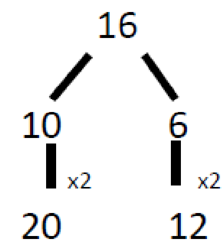
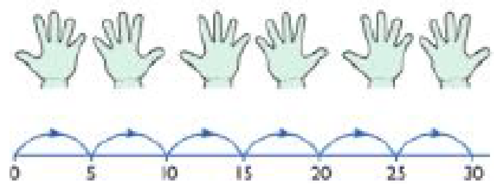
Moving forward the children use a more compact method.

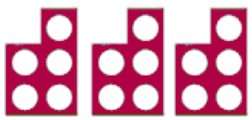
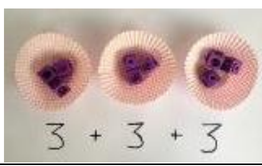
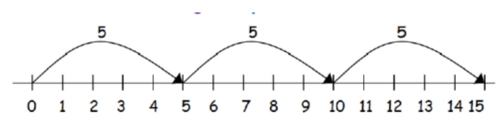
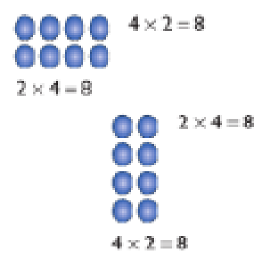
This will lead to an understanding of subtracting any number including decimals.

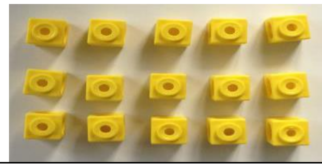
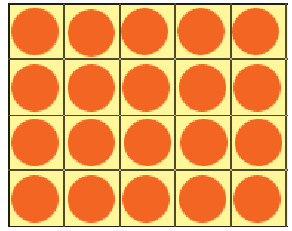
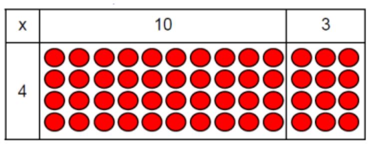
$$
\begin{array}{cccccc}
 & & ^{5} & ^{12} & & ^{1} \\
 & 2 & \cancel{6} & \cancel{3} & . & 0 \\
- & & 2 & 6 & . & 5 \\
\hline
 & 2 & 3 & 6 & . & 5
\end{array}
$$

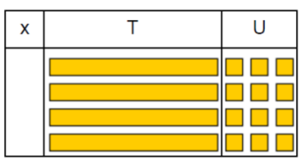
	 Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.		

Multiplication

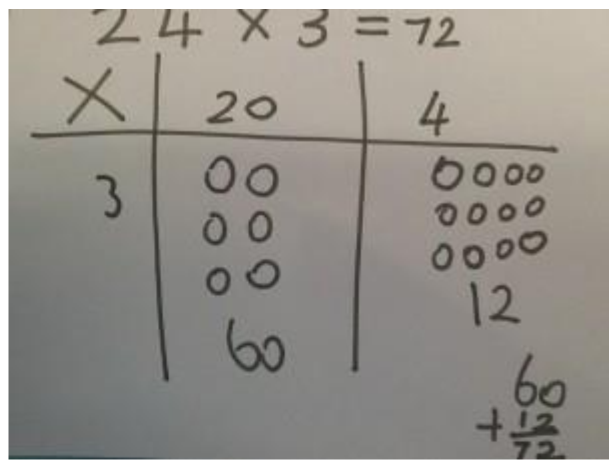
Objective and Strategies	Concrete	Pictorial	Abstract
Doubling	Use practical activities to show how to double a number. 	Draw pictures to show how to double a number. 	 Partition a number and then double each part before recombining it back together.
Counting in multiples	 	 Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

	Count in multiples supported by concrete objects in equal groups.		
Repeated addition	   Use different objects to add equal groups.	  	Write addition sentences to describe objects and pictures. 
Arrays- showing commutative multiplication	Create arrays using counters/ cubes to show multiplication sentences. 	Draw arrays in different rotations to find **commutative** multiplication sentences.  	Use an array to write multiplication sentences and reinforce repeated addition.

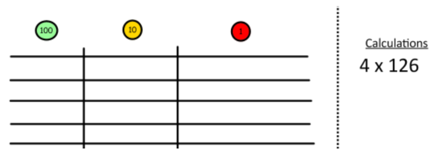
<table>
<tr>
<td></td>
<td></td>
<td>
Link arrays to area of rectangles.</td>
<td>
5 + 5 + 5 = 15
3 + 3 + 3 + 3 + 3 = 15
5 x 3 = 15
3 x 5 = 15</td>
</tr>
<tr>
<td>Grid Method</td>
<td>Show the link with arrays to first introduce the grid method.

4 rows of 10
4 rows of 3

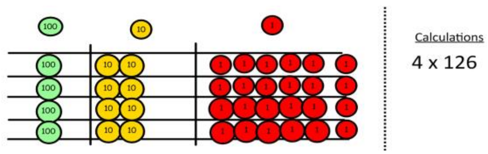
Move on to using Base 10 to move towards a more compact method.

4 rows of 13

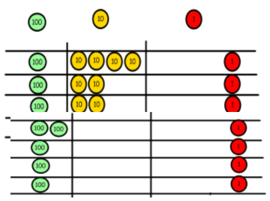
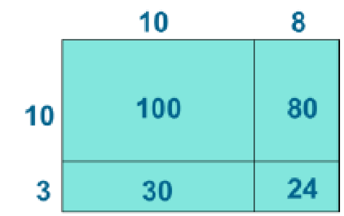
Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.</td>
<td>Children can represent the work they have done with place value counters in a way that they understand.

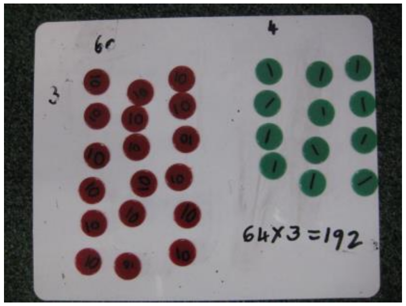
They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.
</td>
<td>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.
<table><tr><td>×</td><td>30</td><td>5</td></tr><tr><td>7</td><td>210</td><td>35</td></tr></table>
210 + 35 = 245

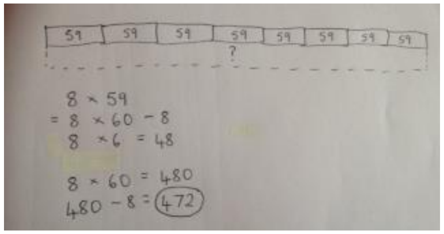
Moving forward, multiply by a 2 digit number showing the different rows within the grid method.</td>
</tr>
</table>

<table>
<tr>
<td></td>
<td>


Fill each row with 126.


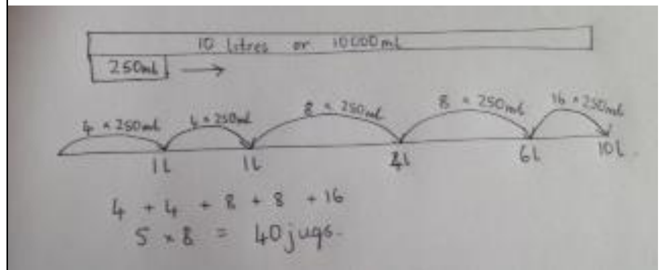
Add up each column, starting with the ones making any exchanges needed.

Then you have your answer.
</td>
<td></td>
<td>


<table>
<tr><th>X</th><th>1000</th><th>300</th><th>40</th><th>2</th></tr>
<tr><td>10</td><td>10000</td><td>3000</td><td>400</td><td>20</td></tr>
<tr><td>8</td><td>8000</td><td>2400</td><td>320</td><td>16</td></tr>
</table>
</td>
</tr>
<tr>
<td>Column multiplication</td>
<td>
Children can continue to be supported by place value counters at the stage of multiplication.


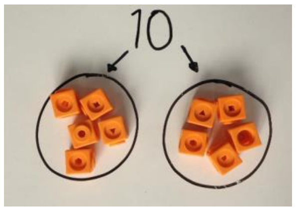
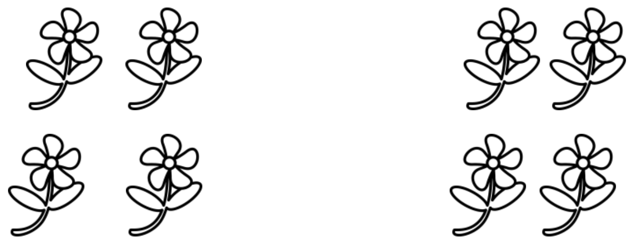
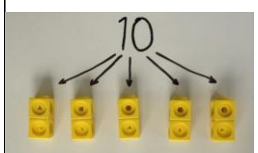
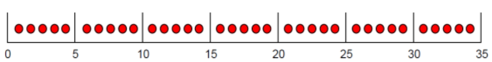
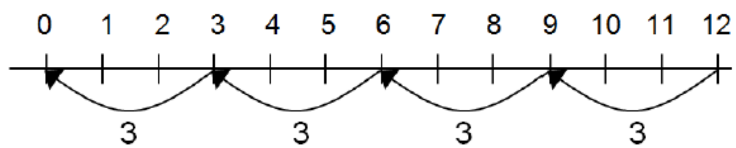
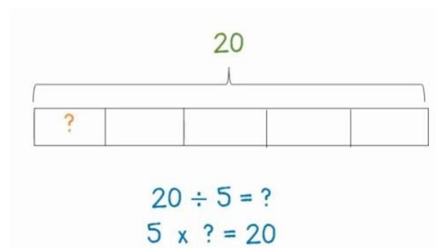
</td>
<td>
Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.


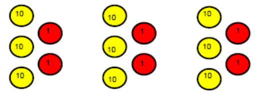
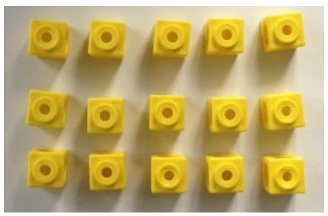
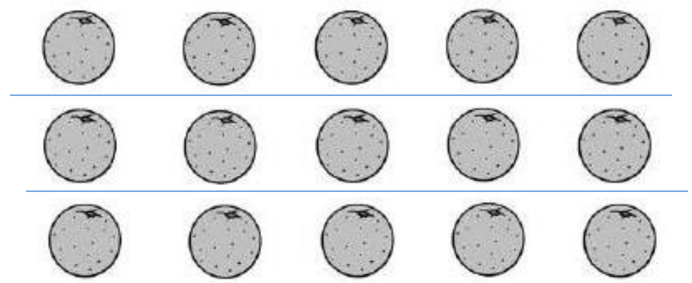
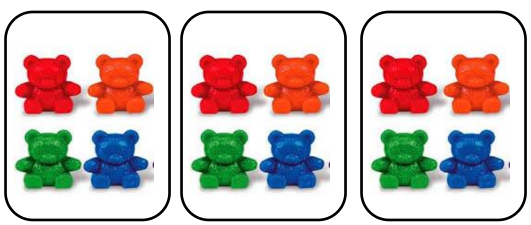
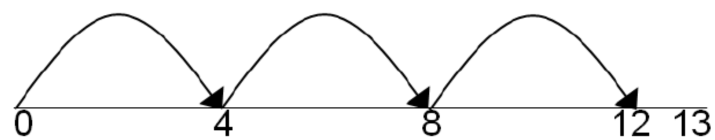
</td>
<td>
Start with long multiplication, reminding the children about lining up their numbers clearly in columns.

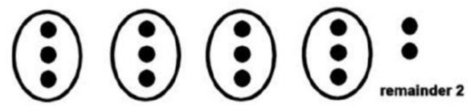
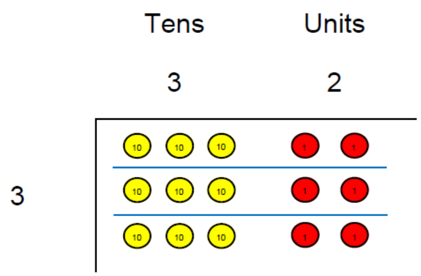
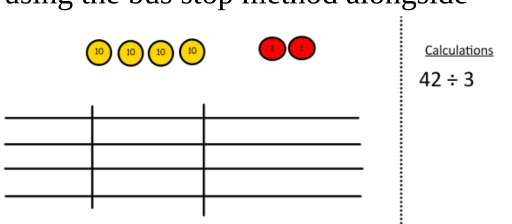
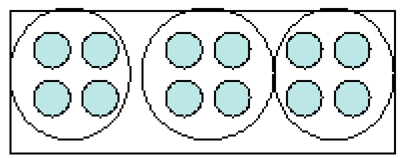
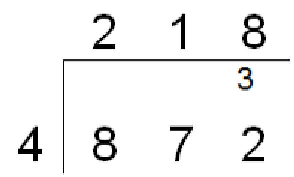
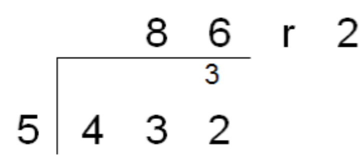
If it helps, children can write out what they are solving next to their answer.
</td>
</tr>
</table>

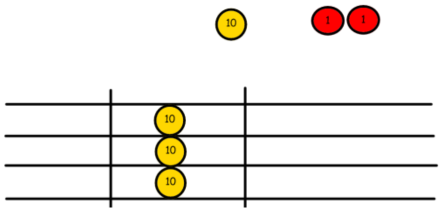
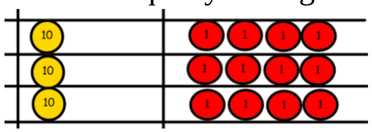
	It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.		32 x 24 8 (4 x 2) 120 (4 x 30) 40 (20 x 2) 600 (20 x 30) 768 74 x 63 12 210 240 + 4200 4662 This moves to the more compact method. 2 3 1 1342 x 18 13420 10736 24156 1

Division

Objective and Strategies	Concrete	Pictorial	Abstract
Sharing objects into groups	 I have 10 cubes, can you share them equally in 2 groups?  	Children use pictures or shapes to share quantities.  $8 \div 2 = 4$	Share 9 buns between three people. $9 \div 3 = 3$
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.   	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.  	$28 \div 7 = 4$ Divide 28 into 7 groups. How many are in each group?

	96 ÷ 3 = 32 		
Division within arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7
Division with a remainder	14 ÷ 3 = Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder. 	Complete written divisions and show the remainder using r. 29 ÷ 8 = 3 REMAINDER 5 dividend divisor quotient remainder

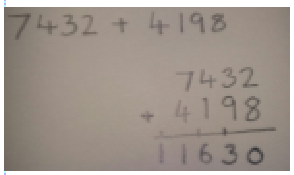
		Draw dots and group them to divide an amount and clearly show a remainder.  	
Short division	 Use place value counters to divide using the bus stop method alongside  42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder.  Move onto divisions with a remainder. 

	 We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.		Finally move into decimal places to divide the total accurately. 1 4 . 6 16 21 3 5 | 5 1 1 . 0

Arithmetic

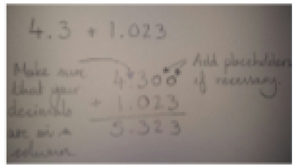
Addition

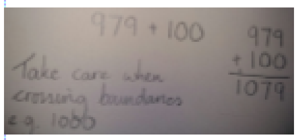
Ensure that you use your place value to arrange your columns accurately.



When adding values you may need to carry. If the values in a column add up to 10 or more you will need to carry.
Example: In the units column 2 and 8 equal 10 so 0 goes in the units column and the 1 is carried.

If the values you are adding have a different amount of decimal places, ensure that the decimal point is in a column.
If you need to, add placeholders into empty columns.

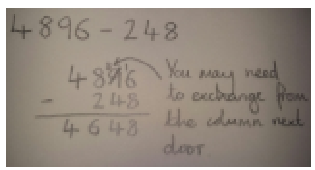


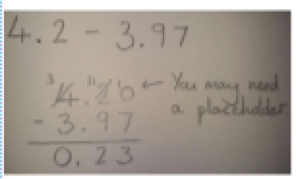


Some addition may be done mentally; however it is also fine to use columns. Watch out for crossing boundaries as they can lead to careless errors.

Subtraction

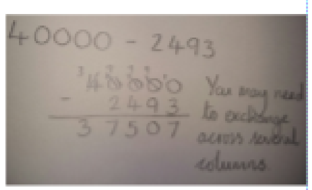
When subtracting, if the number on the bottom is larger than the one on the top, you may need exchange from the column next door.
If the column has a 9 you will change it to an 8 and carry 1 down.





If the number you are subtracting has more decimal places than the one you are subtracting from you will need to add placeholders and exchange.

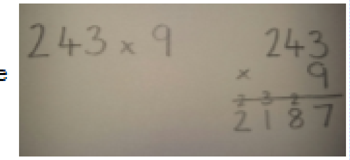
If you are subtracting from a number with lots of zeroes you will need to exchange across several columns.
Example: The 4 in 40000 has been changed to a 3 and the column next door is changed to 10. This is then changed to a 9 and 1 is exchanged to the column next door.

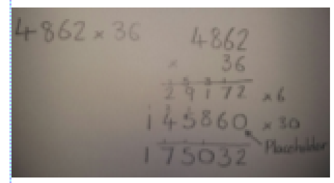


Multiplication

One of the most obvious questions on your SATs will be a straight tables problem e.g. 7 x 8 or 11 x 12

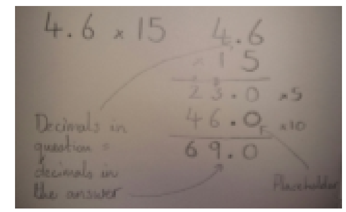
When multiplying values we can use columns. In this case we would multiply each part of 243 by 9. If the answer is greater than 10 we carry to the column next door.

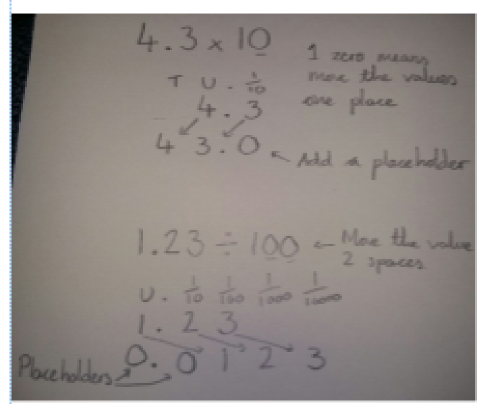




If you are multiplying by a two-digit number, you multiply by the units and then the tens column. When multiplying by the tens column we add a zero first. Finally, we add them together.

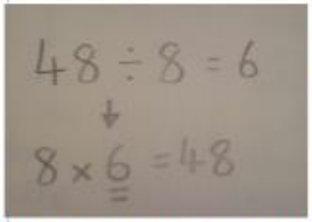
When multiplying a whole number by a decimal you need to place decimal points in the answers in the same place as the question.
Don't forget that we still need a place holder when multiplying by tens.





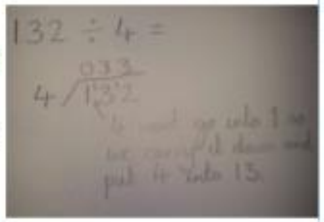
If you are multiplying or dividing by 10, 100 or 1000 you will be moving the digits.
Write the number you are dividing (the dividend) under the correct place value.
Count the number of zeroes in the number you are dividing by (the divisor). Then move the dividend this many places.
If it is multiplying the value gets bigger; if it is division it gets smaller.
You may need to add place holders in any empty spaces.

Division

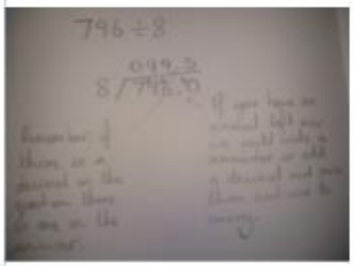


For some division problems we can use our multiplication knowledge to solve the problem. For example, we know that 48 ÷ 8 = 6 because we know that 8 x 6 = 48.

If we are dividing a larger number by a single digit we can use the short division (bus stop) method. We write our dividend (the number being divided up) in the bus stop and our divisor outside. We then try to put the divisor into each value in turn. If the divisor goes into the value we write the amount of times above and then carry any left over to the next column.



In the example 4 didn't go into 1 so we put a zero and carried the 1 down. 4 into 13 three times with one left over. Therefore we wrote 3 above and carried one down again.



Sometimes we are left with some of our original value which can't be divided up any further using whole numbers. For example if I was dividing 7 by 4 , I would have 3 left over. This can be written as a remainder or we can convert into a decimal.

We do this by adding a decimal point and a zero to our dividend and then carrying the remainder into this column.

In this example the remainder was 4; however we carried it down to become 40 which is a multiple of our divisor, 8, so we can divide.

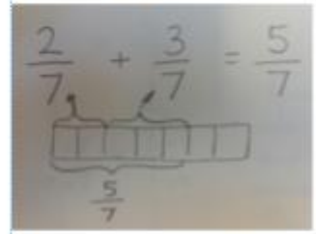
Long division is needed when we are dividing by 2-digit numbers or more. To help out we write a 'cheat sheet' for the divisor e.g. the 66 times tables in our example.

We then use this to help divide up the number. If it will not divide (e.g. 66 cannot go into 5) we write zero above then use the 8 with the 5 to make 58. If the divisor still will not go in we continue this process until it will.

Once we can divide the value (e.g. 66 goes into 587) we use our cheat sheet. 8 x 66 is 528 so we subtract that from 587. This leaves 59. We then pull the 4 down from the next column and continue.

This process can be tricky and it takes time so you need to concentrate.

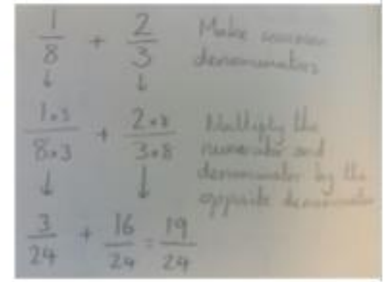
Working with fractions

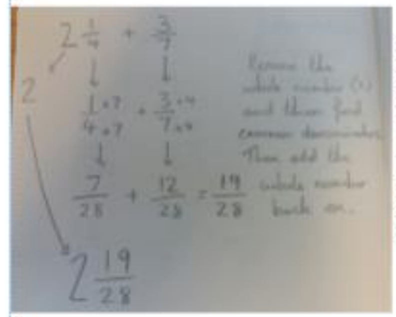


If you are adding fractions with common denominators, you simply add the numerators. For example 3/4 of a pie added to 1/4 of a pie is 4/4 o the pie.

If your denominators are not the same, you can force them to be common by multiplying them by one another. Just remember that whatever you do to the denominator, you also do to the numerator.

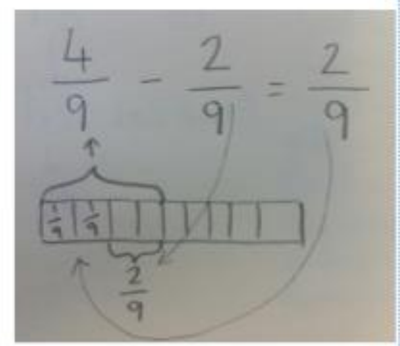
Once the denominators are the same, they can be easily added using the method above.



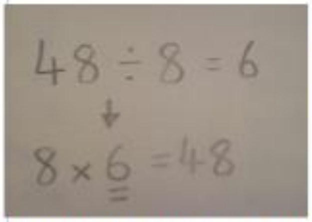


Sometimes you will be working with mixed number (e.g. one and a half) and a fraction. If you are adding and they do not have common denominators, simply remove the whole number (2) for now. Use the methods above to force common denominators and add the fractions. Finally bring the 2 wholes back and add them on.

Subtracting fractions with common denominators is just like adding: we simply work with the numerators.



Division



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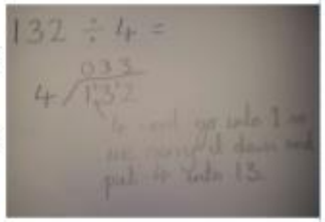

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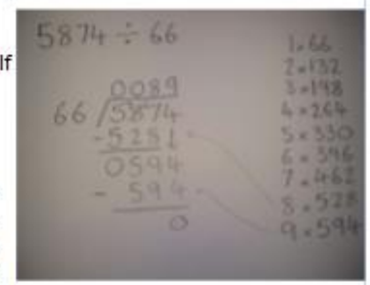
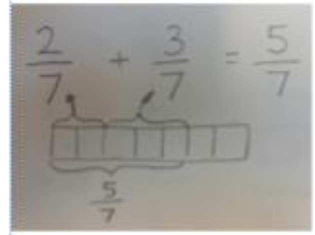


Working with fractions



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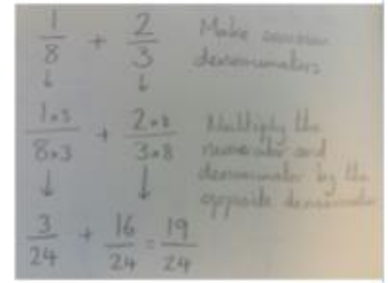
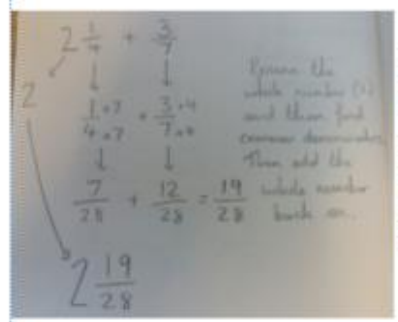
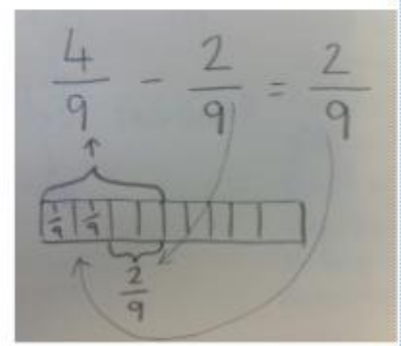
Once the denominators are the same, they can be easily added using the method above.



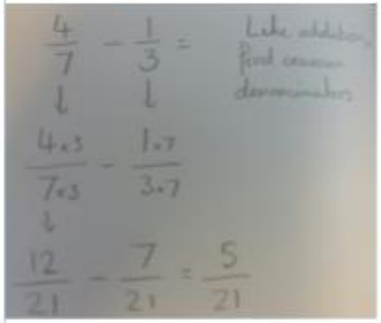


Sometimes you will be working with mixed number (e.g. one and a half) and a fraction. If you are adding and they do not have common denominators, simply remove the whole number (2) for now. Use the methods above to force common denominators and add the fractions. Finally bring the 2 wholes back and add them on.

Subtracting fractions with common denominators is just like adding: we simply work with the numerators.

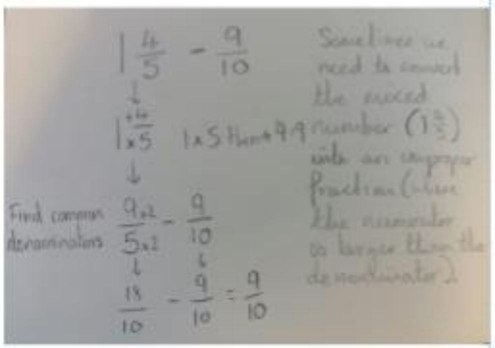


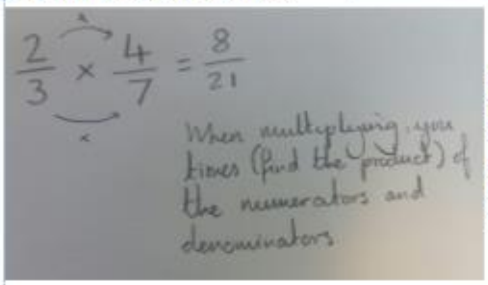
Working with fractions



Subtracting fractions is also like adding given that we force common denominators, then we work with the numerators.

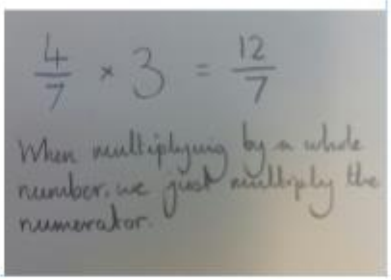
If you are working with mixed numbers (whole numbers and fractions e.g. one and a half) you may need to convert the mixed number into an improper fraction (where the numerator is greater than the denominator). We do this by multiplying the whole number by the denominator and then adding the numerator. We can then force common denominators and calculate.





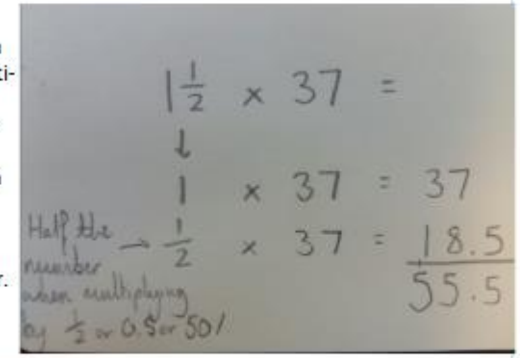
Multiplying to fractions by one another is nice and straight forward. We simply multiply the numerators by one another and do the same with the denominators.

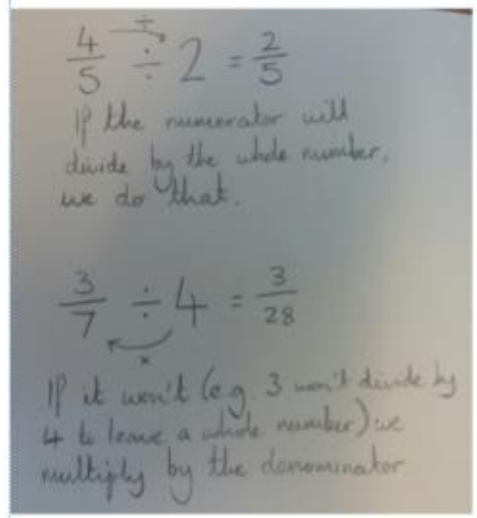
When you multiply by a whole number, simply multiply the numerator by the whole number.



Working with fractions

Multiplying a mixed number can be trickier. However, simply partition the mixed number into its whole number and fraction and multiply each individual. Remember that if you multiply a number by a half it halves the number e.g. 10 x 1/2 = 5 Once you have multiplied both parts, add the answers together.

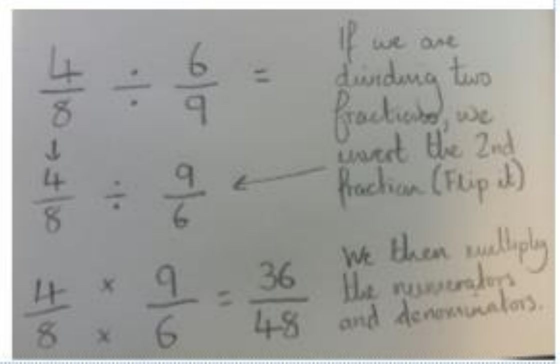




When dividing fraction by a whole number it all depends on whether the numerator is a multiple of the divisor. If it is, you just divide it e.g. in the example 4 is a multiple of 2 so it will divide.

If it will not (as in example 2) then we multiply the denominator instead.

Dividing two fractions can be confusing but relates to our other methods. We inert (flip) the second fraction. Then we multiply the numerators and denominators.



Finding percentages of numbers

Percentage relates to parts of 100. Sometimes we ned to calculate a parentage using our division methods.

30% of 360

If we need to find a percentage which is a multiple of 10 (e.g. 30%) we will need to find 10%.
We find 10% by dividing the number by 10. This involves moving the digits one place to the right.

360 ÷ 10 = 36

36 = 10%

If we multiply the answer by 3 we will get 30%

36 x 3 = 30%

36 x 3 = 108

30% of 380 = 108

32% of 360

If the value is not multiple of 10 (e.g. 32%) we can find 1% and then multiply by the amount we need (32%)

360 ÷ 100 = 3.6

3.6 x 32 = 115.2

Alternatively, we could partition the percentage. We can work out parts and add them together

10%	=	36
10%	=	36
10%	=	36
1%	=	3.6
1%	=	3.6
32%	=	115.2

Finding fractions of numbers

We can find fractions of whole numbers by using the numerator and denominator.

3/5 of 60

First we divide the amount by the denominator.

60 ÷ 5 = 12

Then we multiply by the numerator

12 x 3 = 36

Therefore 3/5 of 60 = 36

Finding the mean

The mean is another word for the average. We find the mean of a set of data by adding the values together and dividing by how many values there were.

For example, here is a set of data containing 5 values.

3 4 6 2 5

Added together they total 20. We divide this total by 5.

20 ÷ 5 = 4

So the mean average of the data is 4.

BIDMAS/BODMAS

Some calculations need to be done in specific order.

B = Brackets
I/O = Indices or orders e.g. 4 squared
D = Division M = Multiplication (Done from left to right)
A = Addition S = Subtraction (Done from left to right)

Example
2 + 3 x 4 (We do the multiplication before addition)
2 + 12 = 14

3^2 + (4—1) (First we solve the brackets, then the indices)
3^2 + 3
9 + 3 = 12

Summary

- Progression is made when pupils are ready, though age related expectation will be followed throughout the school in line with the National Curriculum.
- The children will cover mathematics in three stages of understanding: fluency, reasoning and problem solving.
- Children should be persuaded to estimate first.
- Always check the answer, preferably using a different method e.g. inverse operation.
- Pay attention to language – refer to actual value of digits.
- Children who make persistent mistakes should return to the method that they can use accurately until ready to move on. They will also be supported by the use of manipulatives and concrete objects.
- Children need to know number and multiplication facts by heart.
- Discuss errors and diagnose problems and then work through problem – do not simply re-teach method.
- When revising or extending to more challenging or larger numbers, refer back to expanded methods. This helps reinforce understanding and reminds children that they have an alternative to fall back on if they are having difficulties.